IMPROVED FORMULATION OF THE SINGULAR INTEGRAL EQUATION TECHNIQUE FOR FINLINES

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Abstract

An improved formulation of the singular integral equation (SIE) technique is presented to determine a complete set of finline normal modes of propagation. The present method allows the accurate determination of any number of finline modes. The convergence for the dominant mode as well as for the higher-order modes versus the series truncation order is shown.

Introduction

The determination of an accurate complete set of finline normal modes is of great importance. As has already been shown in [1], the singular integral equation (SIE) technique [2] is very efficient for determining such a set. Due to the asymptotic vanishing of the coefficients of the constructed series, the final characteristic matrix equation, which is solved for the propagation constants, possesses fast convergence properties. So the set of finline normal modes can be accurately determined from a relatively small-order matrix. The already published results give about 20 modes from a (9×9) matrix ([1] and [3]). However, the accurate determination of still higher-order modes requires larger characteristic matrices. In this paper the formulation of the SIE technique is extended to allow the accurate determination of any number of finline modes.

Basic formulation

Following the standard procedure [1], one arrives at a system of linear homogeneous equations

$$A_m^e + 2X_o \sum_n (P_n^e - 1)\, I_{nm}\, A_n^e + 2X_o\, \beta^2 \sum_n P_n^h\, I_{nm}\, A_n^h = 0$$

$$-2I_{om}\, A_o^h + A_m^e + 2X_o \sum_n (Q_n^h - 1)\, I_{nm}\, A_n^h + 2X_o \sum_n Q_n^e\, I_{nm}\, A_n^e = 0$$

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$$\left\{R_o^h - [(\varepsilon_r+1)K_o^2 - 2\beta^2]\right\}A_o^h + \sum_n [\cos(n\phi_o) + nE_nX_o]R_n^hA_n^h + \sum_n [\cos(n\phi_o) + nE_nX_o]R_n^eA_n^e = 0$$

$$m = 1, 2, 3, \ldots\ldots\ldots \qquad (1)$$

where

ϕ_o/π = The ratio (slot width/housing hight)

and $X_o = \sin^2(\phi_o/2)$

I_{nm} and E_n are given by

$$I_{nm} = \frac{1}{\pi}\int_{-1}^{+1} I_n(\eta)\,\frac{\cos(m\phi)}{\sqrt{1-\eta^2}}d\eta \qquad (2)$$

$$I_{om} = \frac{1}{\pi}\int_{-1}^{+1} \frac{\cos(m\phi)}{\sqrt{1-\eta^2}}d\eta \qquad (3)$$

$$E_n = \frac{1}{\pi}\int_{-1}^{+1} I_n(\eta)\,\frac{\ln[2X_o(1-\eta)]}{\sqrt{1-\eta^2}}\,d\eta \qquad (4)$$

$$E_o = \frac{1}{\pi}\int_{-1}^{+1} \frac{\ln[2X_o(1-\eta)]}{\sqrt{1-\eta^2}}\,d\eta \qquad (5)$$

where

$$I_n(\eta') = \frac{1}{\pi}\int_{-1}^{+1} \frac{\sqrt{1-\eta^2}}{(\eta-\eta')}\,\frac{\sin(n\phi)}{\sin\phi}d\eta \qquad (6)$$

with $\phi = \pi y/b$

The coefficients of A_n^e and A_n^h in (1) vanish asymptotically, so the infinite sum can be truncated behind the first N terms. The system of equations in (1) can then be written as a matrix equation

$$\begin{bmatrix} C \end{bmatrix} \underline{X} = 0 \qquad (7)$$

The order of the characteristic matrix C is 2N+1, its elements depend on the unknown propagation constant β. $\underline{X}$ is a column vector containing the unknown field expansion coefficients A_n^e and A_n^h.

To compute the elements of the characteristic matrix, the integrals in (2)-(6) have to be solved. The integral given by (6) contains a singular integral so its numerical calculation is inefficient. In [1] and [3] the integrals have been solved for N equal to 2, 3, and 4, what corresponds to a matrix order of 5, 7, and 9, respectively. In this contribution, analytical expressions have been obtained for these integrals by direct integration and some recurrence relations. The characteristic matrix can consequently be determined for any order. The elements of the characteristic matrix have been normalized such that they are real functions of β^2 if the structure is lossless. The matrix equation is solved by looking for the values of β^2 which make the matrix C singular. Each of these values are one of the looked for modes.

Results

To illustrate the effect of the matrix order on the determination of the set of finline normal modes, the first 33 modes of a bilateral finline fig (1) have been calculated using a matrix order of 11, 15, 21, and 25, respectively. The values of the normalized propagation constant β/K_0 are given in table (1). The convergence for the dominant mode and for some selected higher-order modes versus the series truncation order N is shown in fig. (2). From these results it is clear that the series can be truncated behind N=12 (which corresponds to a matrix order of 25) for calculating the first 50 finline modes accurately. All the reported results are for a finline with a=2b=7.112mm, s=.5mm, d=.254mm, and ε_r=10.

Conclusions

An improved application of the SIE technique to analyze finlines is presented. The present method allows an accurate determination of any number of finline modes.

References

[1] A. S. Omar and K. Schünemann,"Formulation of the singular integral equation technique for planar transmission lines," IEEE Trans. Microwave Theory Tech., vol. MTT-33, pp. 1313-1321, 1985.

[2] L. Lewin, Theory of Waveguides. London: Neurnes Butterworths, 1975.

[3] A. S. Omar and K. Schünemann," Space-domain decoupling of LSE and LSM fields in generalized planar guiding structures," IEEE Trans. Microwave Theory Tech., vol. MTT-32, pp. 1626-1632, 1984.

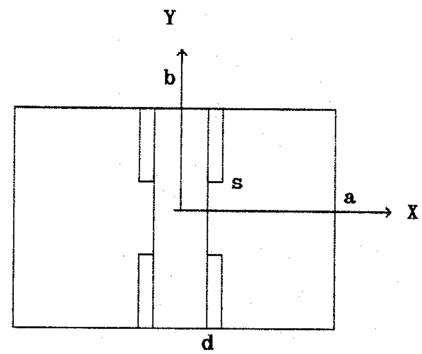


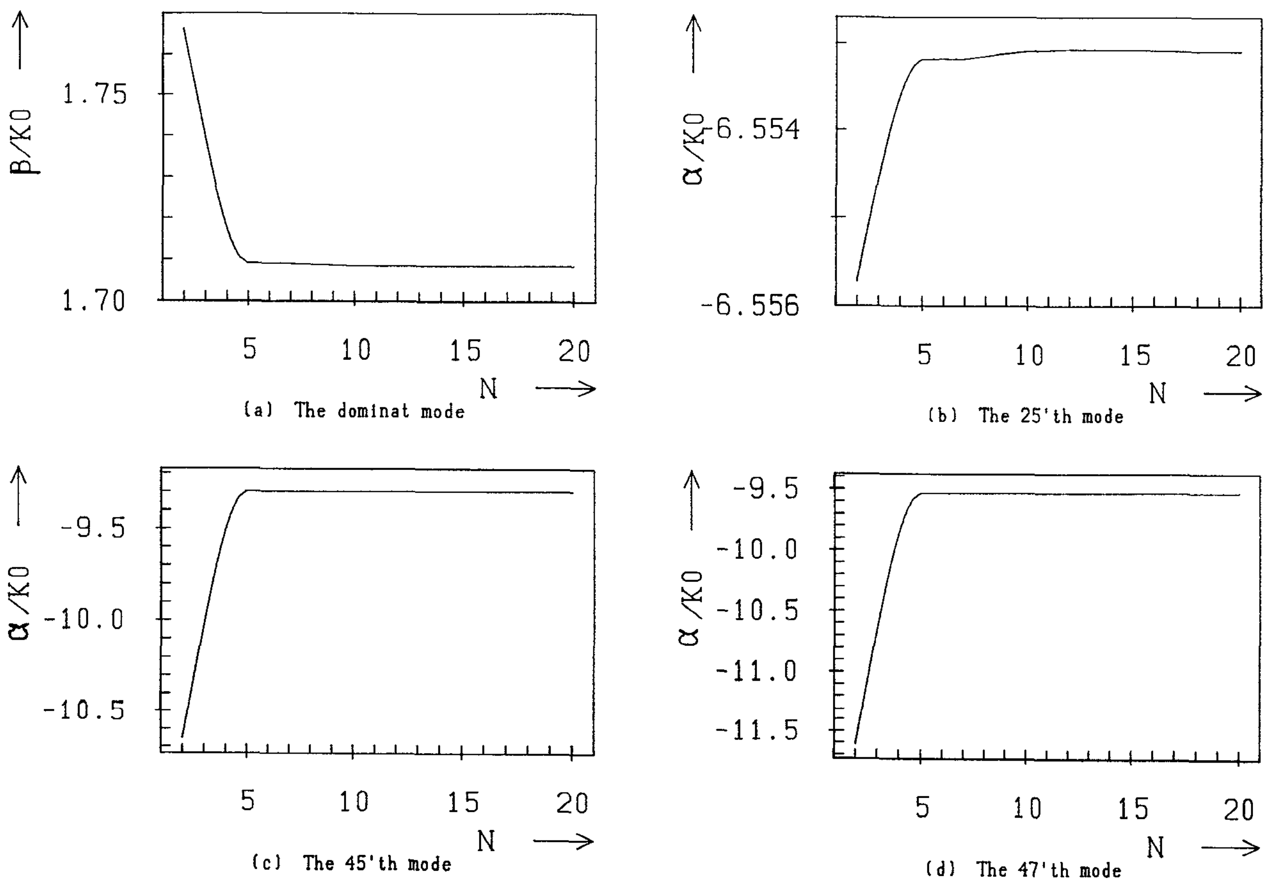
Fig. (1) The cross-section of a bilateral finline.

Table (1)
The propagation constants for the first 33 modes in a bilateral finline

matrix order	mode number 1	2	3	4	5	6	7	8	9	10	11
11	1.70922	-j1.06042	-j1.17464	-j2.62834	-j2.65999	-j2.73824	-j2.80291	-j3.00540	-j3.18300	-j3.92496	-j4.03684
15	1.70909	-j1.06042	-j1.17467	-j2.62834	-j2.65999	-j2.73824	-j2.80292	-j3.00540	-j3.18303	-j3.92496	-j4.03685
21	1.70846	-j1.06042	-j1.17477	-j2.62834	-j2.66000	-j2.73824	-j2.80295	-j3.00540	-j3.18309	-j3.92496	-j4.03686
25	1.70842	-j1.06042	-j1.17477	-j2.62834	-j2.66000	-j2.73824	-j2.80295	-j3.00540	-j3.18309	-j3.92496	-j4.03686

matrix order	12	13	14	15	16	17	18	19	20	21	22
11	-j4.25681	-j4.35126	-j5.10166	-j5.19651	-j5.53468	-j5.56120	-j5.72333	-j5.73646	-j5.74386	-j5.87134	-j6.25523
15	-j4.25681	-j4.35128	-j5.10165	-j5.19651	-j5.53468	-j5.56121	-j5.72333	-j5.73647	-j5.74386	-j5.87134	-j6.25522
21	-j4.25681	-j4.35130	-j5.10164	-j5.19650	-j5.53468	-j5.56121	-j5.72333	-j5.73647	-j5.74386	-j5.87131	-j6.25522
25	-j4.25681	-j4.35130	-j5.10164	-j5.19650	-j5.53468	-j5.56120	-j5.72333	-j5.73647	-j5.74386	-j5.87130	-j6.25522

matrix order	23	24	25	26	27	28	29	30	31	32	33
11	-j6.30222	-j6.39508	-j6.55319	-j7.05316	-j7.11339	-j7.21889	-j7.29699	-j7.74698	-j7.82982	-j8.03825	-j8.11969
15	-j6.30222	-j6.39507	-j6.55318	-j7.05312	-j7.11339	-j7.21889	-j7.29697	-j7.74696	-j7.82979	-j8.03820	-j8.11967
21	-j6.30222	-j6.39506	-j6.55309	-j7.05311	-j7.11339	-j7.21889	-j7.29688	-j7.74695	-j7.82968	-j8.03817	-j8.11957
25	-j6.30222	-j6.39506	-j6.55307	-j7.05311	-j7.11338	-j7.21889	-j7.29687	-j7.74695	-j7.82966	-j8.03817	-j8.11955



(a) The dominat mode

(b) The 25'th mode

(c) The 45'th mode

(d) The 47'th mode

Fig. (2) The convergence for the dominant mode and for some selected higher-order modes versus the series truncation order N.